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International Baccalaureate®
Baccalauréat International
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MATHEMATICS
HIGHER LEVEL
PAPER 2

Monday 10 November 2008 (morning)

2 hours

Candidate session number

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INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all of Section A in the spaces provided.
- Section B: answer all of Section B on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

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Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** the questions in the spaces provided. Working may be continued below the lines, if necessary.*

1. *[Maximum mark: 7]*

In a triangle ABC, $\hat{A} = 35^\circ$, $\text{BC} = 4 \text{ cm}$ and $\text{AC} = 6.5 \text{ cm}$. Find the possible values of $\hat{B}$ and the corresponding values of AB.

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2. *[Maximum mark: 5]*

A geometric sequence has a first term of 2 and a common ratio of 1.05. Find the value of the smallest term which is greater than 500.

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3. *[Maximum mark: 7]*

A continuous random variable X has a probability density function given by

$$f(x) = \begin{cases} \dfrac{(x+1)^3}{60}, & \text{for } 1 \le x \le 3, \\ 0, & \text{otherwise.} \end{cases}$$

Find

(a) $\mathrm{P}(1.5 \le X \le 2.5)$; *[2 marks]*

(b) $\mathrm{E}(X)$; *[2 marks]*

(c) the median of X. *[3 marks]*

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4. *[Maximum mark: 6]*

The angle between the vector $\boldsymbol{a} = \boldsymbol{i} - 2\boldsymbol{j} + 3\boldsymbol{k}$ and the vector $\boldsymbol{b} = 3\boldsymbol{i} - 2\boldsymbol{j} + m\boldsymbol{k}$ is 30°.

Find the values of m.

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5. *[Maximum mark: 5]*

(a) Find the set of values of k for which the following system of equations has no solution.

$$x + 2y - 3z = k$$
$$3x + y + 2z = 4$$
$$5x + 7z = 5$$

[4 marks]

(b) Describe the geometrical relationship of the three planes represented by this system of equations. *[1 mark]*

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6. *[Maximum mark: 7]*

(a) Sketch the curve $y = |\ln x| - |\cos x| - 0.1$, $0 < x < 4$ showing clearly the coordinates of the points of intersection with the x-axis and the coordinates of any local maxima and minima. *[5 marks]*

(b) Find the values of x for which $|\ln x| > |\cos x| + 0.1$, $0 < x < 4$. *[2 marks]*

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7. *[Maximum mark: 8]*

(a) Ahmed is typing Section A of a mathematics examination paper. The number of mistakes that he makes, X, can be modelled by a Poisson distribution with mean 3.2. Find the probability that Ahmed makes exactly four mistakes. *[1 mark]*

(b) His colleague, Levi, is typing Section B of this paper. The number of mistakes that he makes, Y, can be modelled by a Poisson distribution with mean m.

(i) If $\mathrm{E}(Y^2) = 5.5$, find the value of m.

(ii) Find the probability that Levi makes exactly three mistakes. *[5 marks]*

(c) Given that X and Y are independent, find the probability that Ahmed makes exactly four mistakes and Levi makes exactly three mistakes. *[2 marks]*

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8. *[Maximum mark: 7]*

If $y = \ln\left(\frac{1}{3}(1+e^{-2x})\right)$, show that $\frac{dy}{dx} = \frac{2}{3}(e^{-y} - 3)$.

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9. *[Maximum mark: 8]*

The population of mosquitoes in a specific area around a lake is controlled by pesticide. The rate of decrease of the number of mosquitoes is proportional to the number of mosquitoes at any time t. Given that the population decreases from 500 000 to 400 000 in a five year period, find the time it takes in years for the population of mosquitoes to decrease by half.

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SECTION B

*Answer **all** the questions on the answer sheets provided. Please start each question on a new page.*

10. *[Maximum mark: 18]*

(a) Write the vector equations of the following lines in parametric form.

$$\boldsymbol{r}_1 = \begin{pmatrix} 3 \\ 2 \\ 7 \end{pmatrix} + m \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$$

$$\boldsymbol{r}_2 = \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} + n \begin{pmatrix} 4 \\ -1 \\ 1 \end{pmatrix}$$

[2 marks]

(b) Hence show that these two lines intersect and find the point of intersection, A. *[5 marks]*

(c) Find the Cartesian equation of the plane Π that contains these two lines. *[4 marks]*

(d) Let B be the point of intersection of the plane Π and the line $\boldsymbol{r} = \begin{pmatrix} -8 \\ -3 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 8 \\ 2 \end{pmatrix}$.

Find the coordinates of B. *[4 marks]*

(e) If C is the mid-point of AB, find the vector equation of the line perpendicular to the plane Π and passing through C. *[3 marks]*

11. *[Total mark: 21]*

Part A *[Maximum mark: 11]*

(a) A box of biscuits is considered to be underweight if it weighs less than 228 grams. It is known that the weights of these boxes of biscuits are normally distributed with a mean of 231 grams and a standard deviation of 1.5 grams. What is the probability that a box is underweight? *[2 marks]*

(b) The manufacturer decides that the probability of a box being underweight should be reduced to 0.002.

(i) Bill's suggestion is to increase the mean and leave the standard deviation unchanged. Find the value of the new mean.

(ii) Sarah's suggestion is to reduce the standard deviation and leave the mean unchanged. Find the value of the new standard deviation. *[6 marks]*

(c) After the probability of a box being underweight has been reduced to 0.002, a group of customers buys 100 boxes of biscuits. Find the probability that at least two of the boxes are underweight. *[3 marks]*

Part B *[Maximum mark: 10]*

There are six boys and five girls in a school tennis club. A team of two boys and two girls will be selected to represent the school in a tennis competition.

(a) In how many different ways can the team be selected? *[3 marks]*

(b) Tim is the youngest boy in the club and Anna is the youngest girl. In how many different ways can the team be selected if it must include both of them? *[2 marks]*

(c) What is the probability that the team includes both Tim and Anna? *[1 mark]*

(d) Fred is the oldest boy in the club. Given that Fred is selected for the team, what is the probability that the team includes Tim or Anna, but not both? *[4 marks]*

12. *[Maximum mark: 21]*

The function f is defined by $f(x) = x\sqrt{9-x^2} + 2\arcsin\left(\frac{x}{3}\right)$.

(a) Write down the largest possible domain, for each of the two terms of the function, f, and hence state the largest possible domain, D, for f. *[2 marks]*

(b) Find the volume generated when the region bounded by the curve $y = f(x)$, the x-axis, the y-axis and the line $x = 2.8$ is rotated through 2π radians about the x-axis. *[3 marks]*

(c) Find $f'(x)$ in simplified form. *[5 marks]*

(d) **Hence** show that $\int_{-p}^{p} \frac{11-2x^2}{\sqrt{9-x^2}}\,dx = 2p\sqrt{9-p^2} + 4\arcsin\left(\frac{p}{3}\right)$, where $p \in D$. *[2 marks]*

(e) Find the value of p which maximises the value of the integral in (d). *[2 marks]*

(f) (i) Show that $f''(x) = \frac{x(2x^2-25)}{(9-x^2)^{\frac{3}{2}}}$.

(ii) Hence justify that $f(x)$ has a point of inflexion at $x = 0$, but not at $x = \pm\sqrt{\frac{25}{2}}$. *[7 marks]*